HEI-16030101020103OO Seat No. ________

M. Sc. (Sem. I) (CBCS) Examination

November / December - 2017

Physics : Paper - CT - 03

(Quantum Mechanics - I) (New Course)

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) **All** questions carry **equal** marks.
(2) Full marks are indicated at the **right** end of each question.
(3) Symbols have their usual meanings.

1 Answer any **seven** of the following : **14**

(a) Prove that $\left[H, a^{+}\right] = \hbar\omega a^{+}$.

(b) Prove that $J_{-} J_{+} = \vec{J}^{2} - J_z^2 - \hbar J_z$.

(c) What is zero-point energy ?

(d) What is the value of $H_0(\xi)$?

(e) Give the relations of rectangular and spherical polar coordinates.

(f) Why WKB approximation is known as semi-classical approximation ?

(g) What is the main application of variation method ?

(h) Using the first order time independent perturbation equation, $\left(E_k - E_m\right) C_k^{(1)} + H'_{km} + W^{(1)} \delta_{km} = 0$; obtain $C_k^{(1)}$ for $k \neq m$.

(i) In the time dependent perturbation theory, $\left|C_m(t)\right|^2$ indicates what ?

(j) Why matrix is used as operators in quantum mechanics ?

2 Answer any **two** of the following :

(a) Solve the following equation for one dimensional harmonic oscillator using power series method, **7**

$$\frac{d^2h}{d\xi^2} - 2\,\xi\,\frac{dh}{d\xi} + h\,(\epsilon - 1) = 0.$$

(b) Explain harmonic oscillator energy spectrum. **7**

(c) Define the operators "a" and "a^{+}". Derive for the oscillator Hamiltonian as, $H = \hbar\,\omega\left(a^{+}\,a + 1/2\right)$. **7**

3 (a) What is coordinate transformation ? Obtain $\vec{L}_z$ in (θ, ϕ) - representation as, $\vec{L}_z = -\,i\hbar\frac{\partial}{\partial\,\phi}$. **7**

(b) By which relations $\vec{e}_r$, $\vec{e}_\theta$ & $\vec{e}_\phi$ are expressed in terms of $\vec{e}_x$, $\vec{e}_y$ & $\vec{e}_z$ and $\vec{e}_x$, $\vec{e}_y$ & $\vec{e}_z$ in terms of $\vec{e}_r$, $\vec{e}_\theta$ & $\vec{e}_\phi$? **7**

OR

3 (a) Using Schrödinger equation in spherical polar coordinates and using separable variable techniques derive the following radial equation, **7**

$$\frac{1}{r^2}\,\frac{d}{dr}\left(r^2\,\frac{dR}{dr}\right) + \frac{2m}{\hbar^2}\left[E - V(r) - \frac{\lambda}{r^2}\right] R = 0$$

(b) For attractive coulomb potential solve the following radial Schrödinger equation, **7**

$$\frac{d^2u_l}{dr^2} + \left[\frac{2m}{\hbar^2}\left\{E - \left(-\frac{C}{r}\right)\right\} - \frac{l(l+1)}{r^2}\right] u_l = 0$$

where, $V(r) = {-C}/{r}$ and $C = ze^2$ (z is number of protons and e is the electronic charge).

4 Answer any **two** of the following :

(a) For doubly degenerate levels in the time independent perturbation theory, prove that the perturbation removes the degeneracy and obtain the following relation, **7**

$$W^{(1)} = \frac{1}{2}\left(h_{11} + h_{22}\right) \pm \frac{1}{2}\left[\left(h_{11} - h_{22}\right)^2 + 4\, h_{12}\, h_{21}\right]^{1/2}.$$

(b) In the time dependent perturbation theory for Fermi Golden rule, consider the following highly peaked function, **7**

$$\sin^2\left[\left(\omega_{mi} \pm \omega\right) t/2\right] \Big/ \left[\left(\omega_{mi} \pm \omega\right)/2\right]^2$$

and apply the property of delta function and derive the following expression,

$$W_{i \to m} = \frac{2\pi}{\hbar}\left|\left\langle \Phi_m \left| H_1 \right| \Phi_i \right\rangle\right|^2 \rho\left(E_m\right) \Big/ E_m = E_i \pm \hbar\omega.$$

The bar indicates the average value over the final states.

(c) Solve the anharmonic oscillator problem using time independent perturbation theory, the Hamiltonian is given as, $H = p^2 \big/ 2m + \frac{1}{2} kx^2 + a x^3 + b x^4$ and prove that energy eigen values are shifted to higher values by the value of $\frac{3}{2}\, b\, \frac{\hbar\,\omega}{K^2}\, E_0$. **7**

5 Write short notes on : (any **two**)

(a) WKB approximation **7**

(b) Variation method **7**

(c) Spherical Harmonics **7**

(d) The raising, lowering and number operators. **7**